

# PHYSICAL REVIEW B

## SOLID STATE

THIRD SERIES, VOL. 7, NO. 2

15 JANUARY 1973

### Simple Compressibility Relation for Solids\*

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(Received 9 August 1972)

Isothermal compression data derived from shock-wave and static-compression measurements on metals exhibit a nearly precise linear relation between the logarithm of the bulk modulus and the specific volume up to volume changes of 40%. As a result, solid isotherms can be accurately fitted or extrapolated in this range by means of two parameter functions of either a Birch or a modified Tait form.

#### I. INTRODUCTION

The isothermal compression curve of metallic solids can be represented in a strikingly simple manner up to specific volume changes of 40% or up to pressures of nearly twice the normal bulk modulus. The observation is based on a more detailed treatment of static-compression measurements of some very soft metals (the alkalis) and isotherms calculated from shock Hugoniot data on a wide variety of metals.

The simple behavior of the isotherms of metals is evident when the logarithm of the isothermal bulk modulus  $B$  is plotted against volume changes  $\Delta V/V_0$  as shown in Fig. 1. The use of volume as the abscissa rather than pressure was suggested by the simple linear dependence previously found for the melting temperature.<sup>1</sup> Compared with the pressure  $P_T(V)$ , the log of the isothermal bulk modulus  $B_T(V)$ , which can be calculated from shock data with virtually the same accuracy as the pressure, is a more appropriate quantity to fit because of its relatively small variation over the large range of shock compression data. For this reason it is easier to recognize the advantages of a particular method of fitting compression data. Furthermore, a good fit to the volume dependence of the bulk modulus will correspond to an even better fit to the pressure along an isotherm (isothermal pressure). Thus, the nearly linear relation observed in Fig. 1 between  $\log_{10} B$  and  $\Delta V/V_0$  shows clearly that an extremely accurate two-parameter fit to isothermal pressures is possible over a range of 40% in volume changes.

The information for Fig. 1 was indirectly obtained from experimental data by special methods in both the cases of static- and dynamic-compression data. These methods are described in Sec. II. Various accurate ways of fitting the curves of Fig. 1 and comparisons with previous methods are made

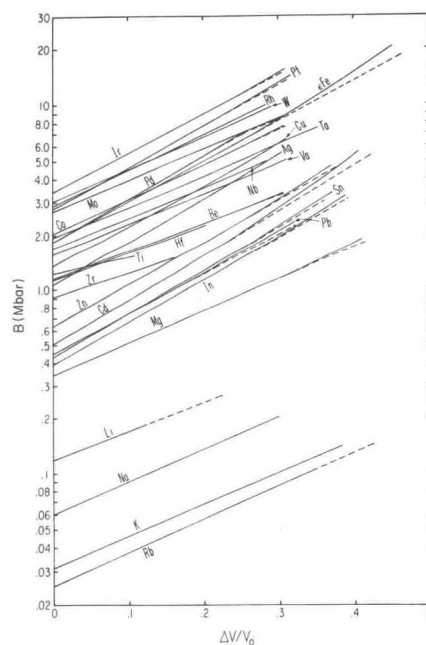


FIG. 1. Isothermal bulk modulus  $B_T$  vs volume change of metals as calculated from shock-wave and static-compression data (see text).

in Sec. III. Finally, the applicability of this behavior to other types of solids is discussed in Sec. IV.

## II. DETERMINATION OF ISOTHERMAL BULK MODULUS FROM COMPRESSION DATA

The bulk of the data exhibited in Fig. 1 is derived from shock-velocity measurements which remains the only method of obtaining compressions in normal solids of 30% or more. These data have previously been shown to be in excellent agreement with static-compression data.<sup>2</sup> Figure 1 contains nearly all the available shock-wave results<sup>3</sup> for metals. In order to reduce the clutter of the graph, a few metals have been left out: Au, Ni, and Cr which are in the Fe group and Re which lies in the W group. Metals for which there is little or no data are pure Al, Mn, Ga, Tc, Ru, and Os plus some heavy elements. Shock data for a number of metals (Na, K, Rb, Hg, Te) lie entirely in the liquid phase, and are not included. In addition a number of metals, in particular the rare earths, the alkaline earths, and first transition elements below Ca and Se, exhibit phase changes at relatively low compressions. Their data have not been included because the range of compression in the low-pressure phases are too small to be of use here. Three metals having low-pressure

phase transitions, Zr, Ti, and Hf, are included.

The 0-deg isotherms plotted in Fig. 1 are derived from shock-velocity data by standard methods.<sup>4</sup> Shock-velocity measurements on solids can nearly always be accurately fit by a linear relation between shock velocity  $U_s$  and material velocity  $U_p$ ; that is  $U_s = C + SU_p$ . The pressure and modulus along the Hugoniot,  $P_H$  and  $B_H$ , are then

$$P_H = \rho_0 U_s U_p = \frac{B_0 x}{(1 - sx)^2}, \quad x \equiv \frac{\Delta V}{V_0}, \quad (1)$$

$$B_H \equiv -V \frac{dP_H}{dV} = B_0 \frac{(1-x)(1+sz)}{(1-sx)^3}, \quad (2)$$

where  $\rho_0$  and  $B_0$  are the normal density and bulk modulus of the solid. Assuming that the solid obeys a Mie-Grüneisen equation of state and further that Grüneisen's  $\gamma$  is related in one of several plausible ways to the shape of the 0-deg isotherm  $P_K(V)$ , the Hugoniot  $P_H(V)$  then determines the 0-deg isotherm. Using standard theories, values of  $B_K(V)$  were calculated for Mg as a typical example and compared with  $B_H(V)$  as shown in Fig. 2. Nominal error bars in the experimental determination of  $B_H(V)$  are also drawn to show that the differences between  $B_K$  and  $B_H$  become significant beyond 20% compression. The theory for  $\gamma(V)$  most often chosen to derive equation-of-state

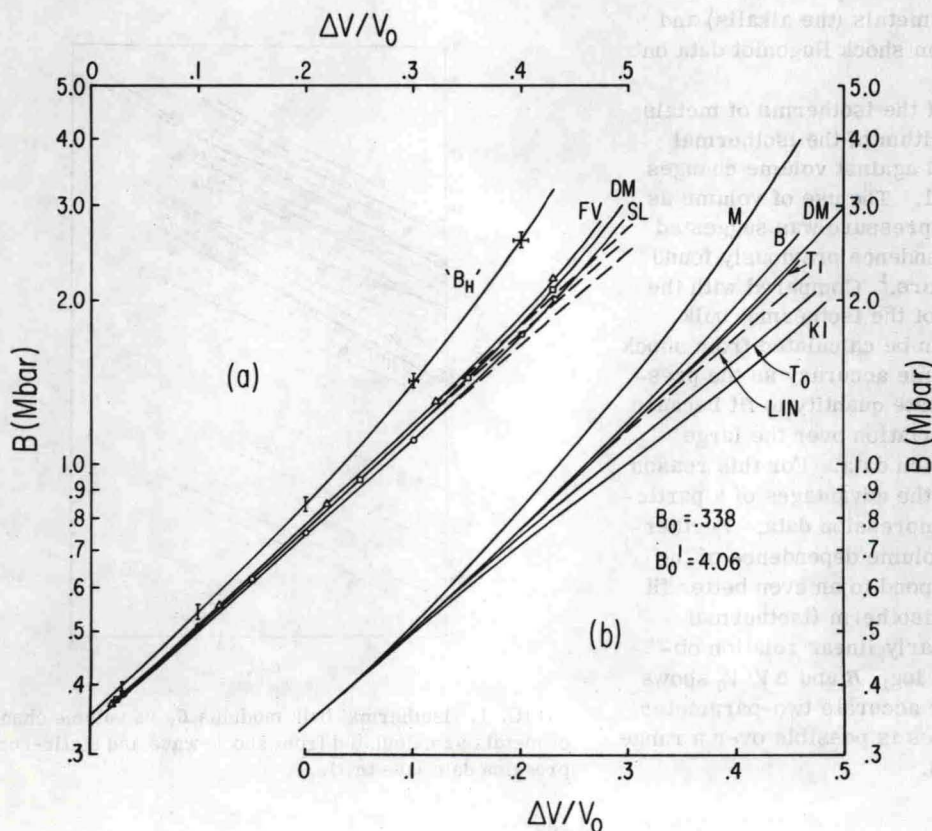


FIG. 2. Comparison of volume dependence of bulk modulus for Mg according to various theories. (a) Hugoniot modulus  $B_H$  compared with bulk modulus on 0-deg isotherm calculated according to various theories (see text). Free-volume theory (FV), Dugdale-MacDonald theory (DM), and Slater-Landau theory (SL). (b) Comparison of DM bulk modulus with the Murnaghan (M), Birch (B), modified Tait ( $T_1$ ), Tait ( $T_0$ ), 2-parameter Keane ( $K_1$ ), and the linear volume relation (see text).